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Optimal Design of Discrete Time Preview Controllers for Semi-Active and Active Suspension systems

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In this paper, modified discrete time preview control algorithms for active and semi-active suspension systems are derived based on a simple mathematical 4 DOF half-car model. The discrete time preview control laws for ride comfort are employed in the simulation. The algorithms for MIMO system contain control strategies reacting against body forces that occur at cornering, accelerating, braking, or under payload, in addition to road disturbances. Matlab simulation results for the discrete time case are compared with those for the continuous time case and the appropriateness of the discrete time algorithms are verified by the of simulation results. Passive, active, and semi-active system responses to a sinusoidal input and an asphalt road input are analysed and evaluated. The simulation results show the extent of performance degradation due to numerical errors related to the length of the sampling time and time delay.

Key Words : Discrete Time, Preview Control, Body Force, Attitude Control

Nomenclature

a : Distance from right suspension mounting point to mass center.
b : Distance from left suspension mounting point to mass center.
b_r, b_l : Damping coefficients of suspensions
f_1, f_2 : Real or hypothetical body forces
I : Moment of inertia
k_{r1}, k_{l1} : Spring constants of suspensions
k_{r2}, k_{l2} : Spring constants of tires
M : Mass
m_r : Mass of right side axle with tire
m_l : Mass of left side axle with tire

Subscript
l : left side
r : right side

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1. Introduction

Applications of advanced technologies to automobile suspension systems contribute to improve the qualities of vehicle dynamic motion with respect to ride comfort, safety in handling, and maintaining the height of the car body. Various innovative types of suspension systems have been studied for the purpose of improving vehicle dynamic motion. When a new suspension system is to developed, an important consideration is to keep the effective distance of the suspension stroke constant regardless of the passengers' weight, payload, or aerodynamic forces in order to preserve the designed suspension functions. The implementation of active or semi-active suspension systems for practical production and actual usage needs to satisfy various requirements, such as sensors with precise measurements, actuators with fast response, advanced control technologies, low cost, among others. A vehicle traveling on an ordinary straight road usually requires negative force to absorb the force impacted by road elevation. The variable damper in a semi-active suspension system which produces only negative force is able to improve the ride characteristics in vehicle dynamics as well as the active system does, because the actuator mainly needs absorption force while the vehicle is traveling on the normal straight road. However, the vehicle when accelerating, braking, or cornering has to react against tilting forces or moments. The reaction force to control the attitude motion of the vehicle is the positive force that supplies energy. These facts mentioned above mean that the capabilities of semi-active suspensions controlled by variable dampers are limited because the attitude motion control of a vehicle needs energy, whereas variable dampers do not.

In this study, the approach is as follows. First, the discrete time preview control algorithm is selected as the most reasonable preview control algorithm for the purpose of real time simulation in the HiL (Hardware-in-the-Loop) system. The discrete time preview control algorithm for SISO systems is derived in (Tomizuka, 1976), while the algorithm in this study is derived differently for applications to MIMO systems. Secondly, simulation using the algorithm applied to a half car model, a MIMO system, is performed and analyzed. The computer simulation results on a front half-car model with active or semi-active suspensions through the application of attitude control and MIMO control which cannot be investigated by a quarter car simulation are evaluated and analyzed. The various computer simulations are carried out by means of the continuous time algorithm and the discrete time algorithm, and the consistency is proved by the comparison of one result with the other, justifying the derived discrete time preview control algorithm. Passive, active, and semi-active system responses to a sinusoidal road input and an asphalt road input are compared and evaluated. In addition, the effect of preview information usage is also evaluated. The observation of the system performance error according to the length of the sampling time shows that the length of the sampling time is limited to 1 msec to neglect the numerical error and the time delay caused by hardware or computation time.

2. Formulation and Controller Design for the Continuous Time Active System

For the computer simulation, a 4 DOF mathematical simple model is utilized. When passengers or payload are loaded or the car is cornering, accelerating, or braking, the real or hypothetical body forces, f_1 and f_2, applied to mounting points occur. For instance, the body forces at cornering can be calculated by the vehicle speed and the angle of the steering wheel. The dynamic equations of motion for the given system are as follows :

$$\begin{aligned} M\ddot{z}_c &= f_1 + f_r + f_2 + f_l \\ I\ddot{\theta} &= (f_1 + f_r)\,a + (f_2 + f_l)\,b \\ m_r\ddot{z}_1 &= -k_{r2}(z_1 - z_{01}) - f_r \\ m_l\ddot{z}_2 &= -k_{l2}(z_2 - z_{02}) - f_l \end{aligned} \tag{1}$$

where

$$\begin{aligned} f_r &= k_{r1}(z_1 - z_c - a\theta) + b_r(\dot{z}_1 - \dot{z}_c - a\dot{\theta}) + u_1 \\ f_l &= k_{l1}(z_2 - z_c + b\theta) + b_l(\dot{z}_2 - \dot{z}_c + b\dot{\theta}) + u_2 \end{aligned}$$

For the actual implementation, the system dynamic states that must be measured or estimated have to be selected by the control designer after considering the possible sensors applicable to the system and the control variables. In this research, the state vector and the disturbance input vector which contains road velocities and body forces applied to mounting points are defined as

$$\begin{aligned} \mathrm{x} &= [z_c + a\theta - z_1,\ \dot{z}_c,\ z_c - b\theta - z_2,\ \dot{\theta}, \\ &\quad z_1 - z_{01},\ \dot{z}_1,\ z_2 - z_{02},\ \dot{z}_2]^T \\ \mathrm{w} &= [\dot{z}_{01},\ \dot{z}_{02},\ f_1,\ f_2]^T \end{aligned} \tag{2}$$

The performance index (or objective function) to be minimized with respect to ride comfort, handling, and allowable suspension stroke is given as

$$J = \lim_{T\to\infty}\frac{1}{2T}\int_0^T\Big[\dot{x}_2^2 + \rho_1\dot{x}_4^2 + \rho_2 x_1^2 + \rho_3 x_3^2 + \rho_4 x_5^2 + \rho_5 x_7^2 + \rho_6 u_1^2 + \rho_7 u_2^2\Big]dt \tag{3}$$

The weighting constants ($\rho's$) which are multiplied by the variances of tire deflections, suspension deflections, body center heaving acceleration, and body rolling angular acceleration are determined by the designer based on experience, statistical data about the vehicle performance, and the design requirements and specifications. The selection of weighting factors are also related to road surface condition, vehicle speed, whether, and the selection of the differential equations required to express the vehicle dynamic motion. The linear state space vector equation for the system with inputs, such as road elevation velocities, body forces, and control forces, is represented as follows :

$$\dot{x}=Ax+Bu+Dw \quad (4)$$

Using the above Eps. (1), (4) and the definition of the state vector (2), the objective function can be expressed as a quadratic form involring the state, disturbance, and control force vectors.

$$J=\lim_{T\to\infty}\frac{1}{2T}\int_0^T\{x^TQx+2x^TN_1u+2x^TN_2w+2w^TM_1u+w^TM_2w+u^TRu\}dt \quad (5)$$

The control force vector which minimizes the objective function is composed of three parts which are calculated by the measurements of the system states, road elevation velocities passed, and the following :

$$u_o(t)=-R^{-1}[(N_1^T+B^TP)x+M_1^Tw_1+B^Tr] \quad (6)$$

where the positive definite solution P is obtained by solving the following Algebraic Riccati equation numerically :

$$PA_n+A_n^TP-PBR^{-1}B^TP+Q_n=0 \quad (7)$$

and

$$r(t)=\int_0^{t_p}e^{A_c^T\sigma}(PD_n+N_n)w(t+\sigma)d\sigma \quad (8)$$

where t_p is the preview time.

The continuous time closed loop system equation is

$$\dot{x}=A_cx+D_nw-BR^{-1}B^Tr \quad (9)$$

where

$$A_c=A-BR^{-1}(N_1^T+B^TP)=A_n-BR^{-1}B^TP$$
$$A_n=A-BR^{-1}N_1^T,\ D_n=D-BR^{-1}M_1^T$$

The optimal control forces consist of a feedback control part $(-R^{-1}(N_1^T+B^TP)x(t))$, reaction forces against the body force $(-R^{-1}M_1^Tw(t))$, and a feed-forward part $(-R^{-1}B^Tr(t))$ which makes use of the preview road or body force information.

Looking at Eq. (6), if there is no preview information, then $r(t)=0$. In that case the feed-forward part is ignored.

3. Formulation and Controller Design for the Discrete Time Active System

At the i-th time instant$(i\Delta t)$, the discrete time state vector and the disturbance input vector are defined as follows :

$$x_i=[z_{ci}+a\theta_i-z_{1i},\ \dot{z}_{ci},\ z_{ci}-b\theta_i-z_{2i},\ \dot{\theta}_i,\ z_{1i}-z_{01i},\ \dot{z}_{1i},\ z_{2i}-z_{02i},\ \dot{z}_{2i}]^T$$
$$w_i=[\dot{z}_{01i},\ \dot{z}_{02i},\ f_{1i},\ f_{2i}]^T \quad (10)$$

where $x_i=x(i\Delta t)$ and Δt is the sampling time.

The Linear state space difference equation for the system is represented by

$$x_{i+1}=A_1x_i+B_1u_i+D_1w_i \quad (11)$$

where the values of the given matrices include the first order terms only in Taylor's series expansion :

$$A_1=I+A\Delta t,\ B_1=B\Delta t,\ D_1=D\Delta t$$

The optimal control laws are derived for general MIMO systems with preview control where we assume that the matrix A_1 is asymptotically stable and preview information about disturbances $\{w_j,\ j\in[i,\ i+N_p]\}$, where $N_p\Delta t$ is the preview time and i represents the present state, can be obtained through estimation or measurements by the preview sensors, such as a sonar sensor, a ultrasonic sensor, a laser beam. The problem given is to determine the optimal control gain which minimizes the objective function J_o within the range from 0 to $N\Delta t$ while satisfying the constraint dynamic equations :

$$J_o=\frac{1}{2}x_N^TP_fx_N+\frac{1}{2}\sum_{i=0}^{N-1}[x_i^TQx_i+2x_i^TN_1u_i+2x_i^TN_2w_i+2w_i^TM_1u_i+w_i^TM_2w_i+u_i^TRu_i] \quad (12)$$

The Hamiltonian is defined as follows :

$$H_i=\frac{1}{2}(x_i^TQx_i+2x_i^TN_1u_i+2x_i^TN_2w_i$$

$$+2w_i^T M_1 u_i + w_i^T M_2 w_i + u_i^T R u_i) + \lambda_{i+1}^T (A_1 w_i + B_1 u_i + D_1 w_i) \quad (13)$$

The objective function (12) can be written using Eq. (13) as follows :

$$J_o = \frac{1}{2} x_N^T P_f x_N + \frac{1}{2}\sum_{i=1}^{N-1} (H_i - \lambda_i^T x_i) + H_0 - \lambda_N^T x_N \quad (14)$$

In a continuous time system, the optimal control force to minimize the objective function is determined at the point satisfying the condition $\frac{dJ}{dt}=0$. However, in a discrete time system, the above condition is approximated by a Taylor's series expansion ; that is, $J(t+\Delta t) = J(t) + \frac{dJ(t)}{dt}\Delta t + \frac{d^2J(t)}{dt^2}\Delta t^2 \cdots$. If the terms higher then first-order are ignored, the following relation $\frac{\Delta J}{\Delta t} = \frac{dJ}{dt}$ is obtained. The variation ΔJ is expressed by Δx_N, Δx_o, Δu_o, Δx_i, Δu_i :

$$\Delta J_o = (x_N^T P_f - \lambda_N^T)\Delta x_N + \frac{\partial H_o}{\partial x_o}\Delta x_o + \frac{\partial H_o}{\partial u_o}\Delta u_o + \sum_{i=1}^{N-1}\left[\left(\frac{\partial H_i}{\partial x_i} - \lambda_i^T\right)\Delta x_i + \frac{\partial H_i}{\partial u_i}\Delta u_i\right] \quad (15)$$

The objective function can be minimized by setting $\Delta J=0$, which has to satisfy the following equalities with initial conditions $\Delta x_o = \Delta u_o = 0$:

$$\lambda_i = \left(\frac{\partial H_i}{\partial x_i}\right)^T,\ \lambda_N = P_f x_N,\ \left(\frac{\partial H_i}{\partial u_i}\right)^T = 0 \quad (16)$$

Condition (16) can be rewritten as

$$\lambda_i = Q x_i + N_1 u_i + N_2 w_i + A_1^T \lambda_{i+1},\ \lambda_N = P_f x_N \quad (17)$$

$$u_i = -R^{-1}(N_1^T x_i + M_1^T w_i + B_1^T \lambda_{i+1}) \quad (18)$$

Since the system is linear, we can assume the following equalities (Hac, 1992; Youn, 1992) :

$$\lambda_i = P_i x_i + r_i,\ \lambda_N = P_f x_N \text{ and } r_N = 0 \quad (19)$$

Substituting Eq. (19) into (17) and (18), and manipulating them yields

$$x_{i+1} = A_{1n} x_i - B_1 R^{-1} B_1^T (P_{i+1} x_{i+1} + r_i) + D_{1n} w_i,\ x_0 = x(t_0) \quad (20)$$

$$\lambda_i = P_i x_i + r_i = Q_n x_i + A_{1n}^T P_{i+1} x_{i+1} + A_{1n}^T r_{i+1} + N_n w_i,\ \lambda_N = P_f x_N \quad (21)$$

where

$$D_{1n} = D_1 - B_1 R^{-1} M_1^T,\ N_n = N_2 - N_1 R^{-1} M_1^T$$

Rewrite Eq. (21)

$$x_i = (P_i - Q_n)^{-1}(A_{1n}^T P_{i+1} x_{i+1} + A_{1n}^T r_{i+1} + N_n w_i - r_i) \quad (22)$$

Plugging (22) into (20) and separating the state variables yields

$$[I - A_{1n}(P_i - Q_n)^{-1} A_{1n}^T P_{i+1} + B_1 R^{-1} B_1^T P_{i+1}] x_{i+1} = [A_{1n}(P_i - Q_n)^{-1} A_{1n}^T - B_1 R^{-1} B_1^T] r_{i+1} - A_{1n}(P_i - Q_n)^{-1} r_i + [A_{1n}(P_i - Q_n)^{-1} N_n + D_{1n}] w_i \quad (23)$$

To satisfy the above equation for arbitrary x_{i+1}, the following two equations must be satisfied :

$$I - A_{1n}(P_i - Q_n)^{-1} A_{1n}^T P_{i+1} + B_1 R^{-1} B_1^T P_{i+1} = 0 \quad (24)$$

and $r_i = A_{1c}^T r_{i+1} + Q_c w_i$ (25)

where

$$A_{1c}^T = A_{1n}^T - (P - Q_n) A_{1n}^{-1} B_1 R^{-1} B_1^T,$$
$$Q_c = N_n + (P - Q_n) A_{1n}^{-1} D_{1n} \quad (26)$$

Assuming that $P_i = P_{i+1} = P$ in steady state, the algebraic Riccati equation can be obtained :

$$P A_{1n}^{-1} - (A_{1n}^T + Q_n A_{1n}^{-1} B_1 R^{-1} B_1^T) P + P A_{1n}^{-1} B_1 R^{-1} B_1^T P - Q_n A_{1n}^{-1} = 0 \quad (27)$$

Observing Eqs. (7) and (27), The form of both equations are identical. Therefore the P matrix can be easily achieved by numerical methods solving the algebraic Riccati equation for the continuous time algorithm.

Arranging Eq. (25) in time series, it can be modified as the following expression :

$$r_i = \sum_{l=0}^{N_p-1} (A_{1c}^T)^l Q_c w_{i+l} \quad (28)$$

where $N_p = t_p/\Delta t$ is the number of sampling intervals by which the preview time is divided. Using the P matrix and vector r_i, the optimal preview control force u_i can be determined as

$$u_i = -R^{-1}[N_1^T + B_1^T A_{1n}^{-T}(P - Q_n)] x_i - R^{-1}(M_1^T - B_1^T A_{1n}^{-T} N_n) w_i - R^{-1} B_1^T A_{1n}^{-T} r_i \quad (29)$$

Substituting the control force (29) into the system dynamic Eq. (11), the closed loop system equation is obtained as follows :

$$x_{i+1} = [A_{1n} - B_1 R^{-1} B_1^T A_{1n}^{-T}(P - Q_n)] x_i + [D_{1n} + B_1 R^{-1} B_1^T A_{1n}^{-T} N_n] w_i - B_1 R^{-1} B_1^T A_{1n}^{-T} r_i \quad (30)$$

Tomizuka's solution for SISO systems is modified as below for the purpose of application to MIMO systems considering body forces. The equations derived by the alternative method can be compared with the equations derived above. The control force (29) determines r_l while the following method requires r_{l+1}.

$$u_l = -(R + B_l^T P B_l)^{-1}[(N_l^T + B_l^T P A_l) x_l + B_l^T r_{l+1} + (M_l^T + B_l^T P D_l) w_l] \quad (31)$$

The P matrix which is required to compute the control forces is solved by algebraic Riccati equation :

$$P = Q_n + A_{ln}^T P A_{ln} - A_{ln}^{-1} P B_l (R + B_l^T P B_l)^{-1} B_l^T P A_{ln} \quad (32)$$

Where the vector r_{l+1} is given by

$$r_{l+1} = \sum_{j=0}^{N_p-l-2} (A_{lc}^T)^{N_p-l-j-2} A_{ln}^T P (I + B_l R^{-1} B_l^T P^{-1}) D_{ln} w_{N_p-j-1} \quad (33)$$

and the closed loop system matrix is

$$A_{lc} = A_l - B_l (R + B_l^T P B_l)^{-1} (N_l^T + B_l^T P A_l) \quad (34)$$

The closed loop system Eq. (35) corresponds to Eq. (30) :

$$x_{l+1} = A_{lc} x_l - B_l (R + B_l^T P B_l)^{-1} B_l^T r_{l+1} + [D_l - B_l (R + B_l^T P B_l)^{-1} B_l^T P D_{ln}] w_l \quad (35)$$

There are two important differences between the two discrete time algorithms. The first one is that the control force (29) computes r_l while Eq. (31) requires the computation of r_{l+1}. The other difference is as follows. The structure of the algebraic Riccati equation for discrete time derived above is the same as that for continuous time. Therefore the solution of the algebraic Riccati equation for the discrete time algorithm can be obtained easily by an eigenvalue eigenvector method or Schur's method.

4. The Determination of Damping Coefficients for the Discrete Time Semi-Active Systems

In the simple model of Fig. 1 drawn for the active system, if not replace the actuator, $u(t)$, by the variable damper, $v(t)$, then it will become a semi-active suspension system model. The bilinear state space difference equation for the dynamic motions of the semi-active system is expressed as

$$x_{l+1} = A_l x_l - B_l (\text{diag } v_l) E x_l + D_l w_l \quad (36)$$

$$\text{where } E = \begin{bmatrix} 0 & 1 & 0 & a & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -b & 0 & 0 & 0 & -1 \end{bmatrix}$$

The continuously variable dampers can control the damping force. Those damping coefficients can vary within certain ranges as follows :

$$-v_l \leq 0, \; v_l - v_{max} \leq 0 \quad (37)$$

The optimal active control force u_l is expressed as a damping force $-(\text{diag } v_l) E x_l$. The strategies to decide the optimal damping coefficients v_{ij} (j = 1, 2), where j = 1, 2 represent the right and left side, respectively, are

$$v_{ij} = \begin{bmatrix} 0 \quad \text{if} \quad \dfrac{-u_{oij}}{\dot{x}_{i(2j-1)} - \dot{x}_{i(2j+1)}} \leq 0 \\ v_{ijmax} \quad \text{if} \quad \dfrac{-u_{oij}}{\dot{x}_{i(2j-1)} - \dot{x}_{i(2j+1)}} \geq v_{ijmax} \\ \dfrac{-u_{oij}}{\dot{x}_{i(2j-1)} - \dot{x}_{i(2j+1)}} \quad otherwise \end{bmatrix} \quad (38)$$

5. Analysis of Simulation Results

The performance of a 4-DOF half car model with passive, active, and semi-active suspension systems is investigated through a Matlab simulation. The data for the parameters shown in Fig. 1 are given below for the passive system and the active system :

$M = 835[kg]$, $I = 910[m \cdot kg]$, $m_r = 46[kg]$,

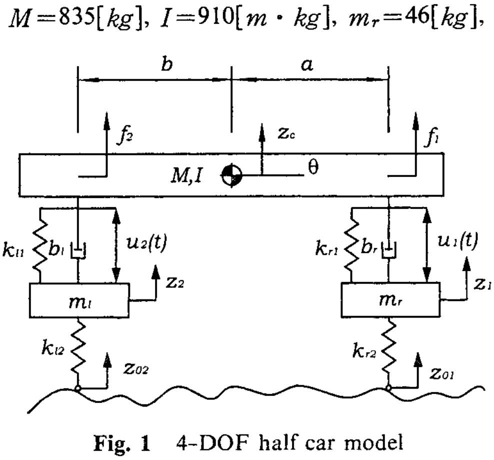
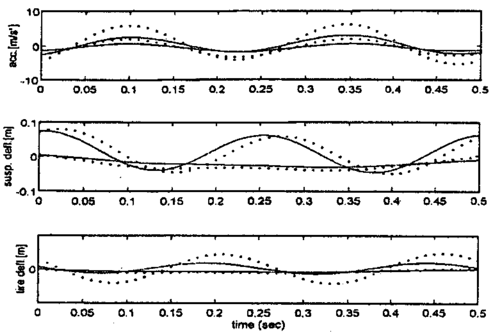
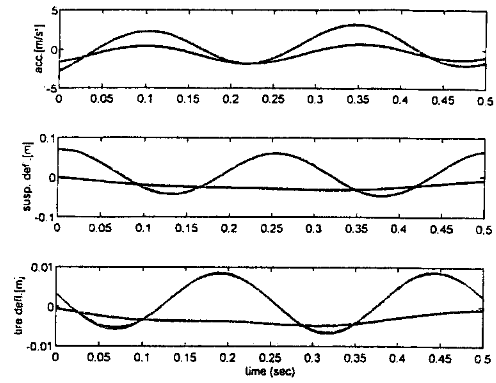


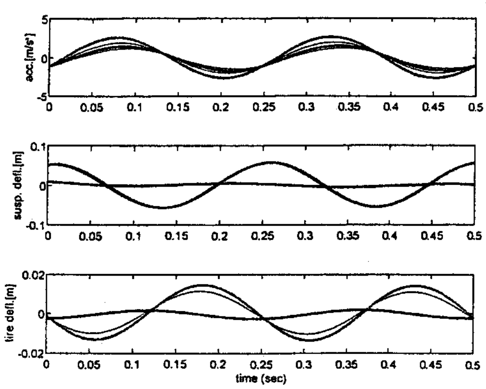
Fig. 1 4-DOF half car model



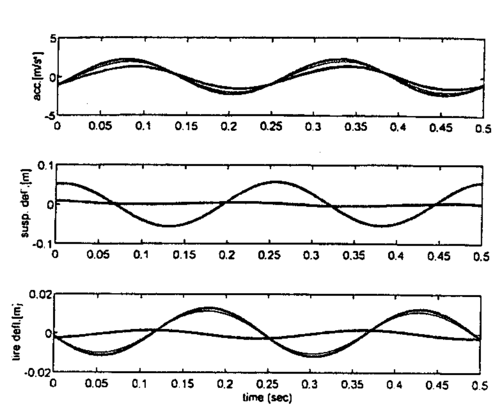
(a) Passive system
(length of sampling time=10 msec.)



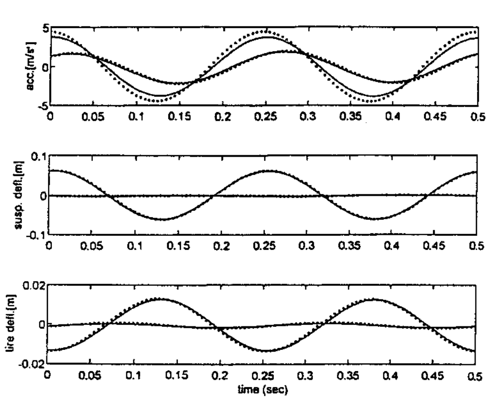
(b) Passive system
(length of sampling time=2 msec.)



(c) Active system
(length of sampling time=2 msec.)



(d) Active system
(length of sampling time=1 msec.)

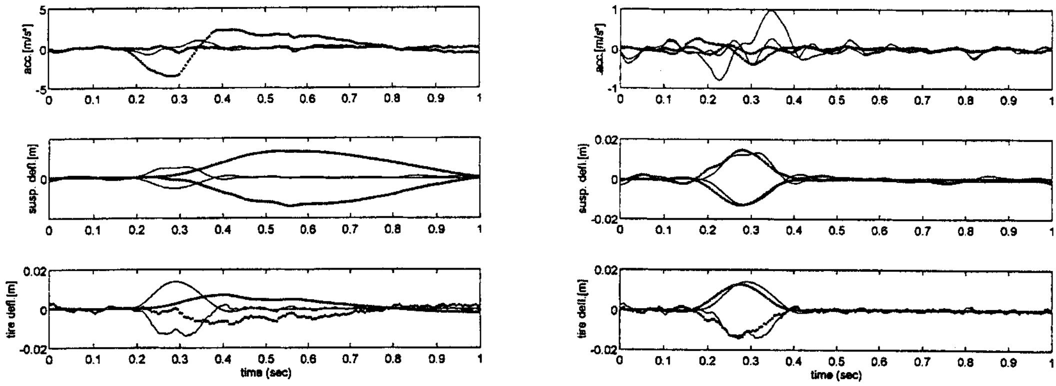


(e) Active system with preview 0.1 sec
(length of sampling time=5 msec.)

Fig. 2 When the sinusoidal road input of frequency 25 rad/sec. is applied to right side wheel, comparison of discrete time with continuous time half car simulation result are shown above

$m_l=46[kg]$, $k_{r1}=k_{l1}=26.4[KN/m]$, $k_{r2}=k_{l2}=184[KN/m]$, $b_r=1000[Ns/m]$, $b_l=1000[Ns/m]$, $a=0.56[m]$, $b=0.56[m]$

In the case of the semi-active suspension system, $b_r=200[Ns/m]$, $b_l=200[Ns/m]$ are used as the minimum damping coefficients. The weighting



(a) -solid line : response of active system
-dotted line : response of semi-active system with continuously variable damper

(b) -solid line : response of active system
-dotted line : response of active system with 0.1 sec. preview road and body force information

Fig. 3 When the asphalt road input and body force are applied to one side wheel and mounting points of car body respectively

factors in the objective function are selected for the purpose of ride comfort as $\rho_1=2$, $\rho_2=\rho_3=1000$, $\rho_4=\rho_5=10000$, $\rho_6=\rho_7=0$. In Fig. 2, the discrete time simulation results according to several intervals of sampling time are compared with the continuous time simulation results and evaluated. The heaving acceleration and rolling angular acceleration are shown in the upper part of (a)-(e) in the Fig. 2. The middle part expresses the deflections of the right and left side suspensions. The deflections of the tires on both sides are shown in the lower part. The tire deflections are related to vehicle safety or handling, which means that the dynamic vehicle is safer for smaller changes in the ground gripping force. The numerical error caused by the length of the sampling time (10 msec) in Fig. 2(a) is almost eliminated as shown in (b) when the simulation of the passive system is performed with 2 msec sampling time. For the active system, a simulation performed with 2 msec sampling time still contains numerical error due to a miscalculated control force shown in (c). However, the response plots in (d) suggest that the 1 msec sampling time makes any discrete time simulation very closely follow the continuous time simulation. As a result, we can conclude that 1 msec is the sampling time limit for which the results are not affected by numerical error. The 1 msec should include any time delay caused by the sensors, actuators, computation, etc. Real time simulation requires one to accomplish the complete computation for one step motion of the dynamic system within one discrete time (1 msec). Recently, HiL systems in which the dynamic simulation code communicates and cooperates with parts of the real system through electrical signals have been studied as real time working problems in various fields. HiL systems help researchers run and evaluate the large systems in small research areas. In Fig. 2(e), the preview control simulation result performed with 5 msec sampling time almost nearly follows the ideal simulation result. The fact that the correct simulation result is obtained for even 5 times larger sampling times with the assistance of preview information shows the potential of preview control.

The same result on discrete time preview control has also been reported in Tomizuka (1976).

In Fig. 3, the road unevenness around the mean value is described by a stationary stochastic process with spectral density

$$S(\omega)=\frac{\sigma^2}{\pi}\frac{\alpha v_o}{\omega^2+(\alpha v_o)^2}$$

where v_o, the forward velocity, was assumed to be 20 m/s, ω is circular frequency, σ the standard deviation of road unevenness and $\alpha=0.15m^{-1}$

and $\sigma^2=9\cdot10^{-6}m^2$ were assumed. The body forces are applied to the mounting points as follows :

$$f_1(t)=-2000b[1-\cos8\pi(t-0.15)]N \ at \ 0.15_s\leq t\leq0.4_s$$
$$f_2(t)=2000a[1-\cos8\pi(t-0.15)]N \ at \ 0.15_s\leq t\leq0.4_s$$

Using those asphalt road input and body forces, the response of the vehicle model was computed. Figure 3(a) shows the vehicle motion response when vehicles with active suspensions and semi-active suspensions are cornering on an asphalt road. The active system responds well to body forces while the semi-active system doesn't overcome the rolling motion because the variable damper cannot supply energy, but can only dissipate energy. This result implies that the limitation of semi-active systems appears in attitude control. The active preview control system in Fig. 3(b) performs extremely well with 0. 1 sec preview road and body force information.

6. Conclusions

Discrete time preview optimal control laws for active and semi-active vehicle suspensions were derived based on a simple half car model. MIMO control algorithms contain preview control strategies to improve the response to body forces occurring during cornering, accelerating, braking, or under payloads in addition to road inputs. These discrete time control algorithms were verified by the similarity between the results of continuous time simulations and those of discrete time simulations with 1 msec sampling time. Two achievements distinguished from the alternative solution are as follows. In this derivation, r_l is required while the other solution reguires the solntion of r_{l+1} and the structure of the algebraic Riccati equation for discrete time solution is the same as that for the continuous time solution. The limited sampling time for the correct numerical result is 1 msec for the given system in this study. However, 5 msec sampling time is enough for preview control simulation. From this research, the fundamental limitations of semi-active suspensions are exposed when the vehicle is subjected to body forces. The simulation results show that the attitude control performance of semi-active suspension system is much worse than that of active suspension systems.

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References

Cho, B., 1999, "Active Suspension Controller Design Using MPC with Preview Information," *KSME International Journal*, (ISSN 1226-4865), pp. 168~174.

Chung, S., and Haug, E. J., 1993, "Real-Time Simulation of Multibody Dynamics on Shared Memory Multiprocessors," *Trans. ASME, Journal of Dynamic Systems, Measurement, and Control*, Vol. 115, pp. 627~637.

Hac, A., Youn, I. and Chen, H. H., 1996, "Control of Suspensions for Vehicles with Flexible Bodies-Part I: Active Suspension, Part II: Semi-Active Suspension," *Trans. ASME, Journal of Dynamic Systems, Measurement, and Control*, Vol. 118, pp. 508~525.

Hac, A., 1992, "Optimal Linear Preview Control of Active Vehicle Suspension", *Vehicle System Dynamics*, Vol. 21, No. 3, pp. 167~195.

Hwang, S., Heo, S., Kim, H. and Lee, K., 1997, "Vehicle Dynamic Analysis and Evaluation of Continuously Controlled Semi-Active Suspensions Using Hardware-in-the-loop Simulation," *Vehicle System Dynamics*, Vol. 27, pp. 423~434.

Lee, K., 1995, "Vehicle Parameter Identification for Active and Semi-Active Suspension Control," *Trans. KSME*, (ISSN 1225-5963) pp. 2505~2513.

Park, Y., Roh, H., 1998, "Observer-Based Wheelbase Preview Control of Active Vehicle Suspensions," *KSME International Journal*, (ISSN 1226-4865), pp. 782~791.

Tomizuka, M., 1976, "Optimum Linear Preview Control with Application to Vehicle Suspension," *Trans. ASME, Journal of Dynamic Sys-*

tems, Measurement, and Control, pp. 309～315.

Tomizuka, M., and Whitney, D. E., 1975, "Optimal Discrete Finite Preview Problems (Why and How is Future Information Important ?)," *Trans. ASME, Journal of Dynamic Systems, Measurement, and Control*, pp. 319～325.

Youn, I., 1992, "Optimal Control of Semi-Active Automobile Suspension Including Preview Information," Ph. D Thesis, State University of New York, Stony Brook, New York.